AN INVESTIGATION OF KALMAN'S AND HOWITT'S EQUIVALENCE TRANSFORMATIONS

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THESIS

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ABSTRACT

Kalman's and Howitt's equivalence transformations are applied to the canonic impedance and admittance Foster LC forms and the Cauer ladder realizations for an RC circuit. The results provide a format for transforming from one realization to another directly. Application of the Kalman transformation to second-order Brune and Bott-Duffin realizations indicate that they are not compatible, implying the incompleteness of Kalman's transformation theory. The same technique is used to show a similar incompleteness of Howitt's theory.

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I. INTRODUCTION

One of the biggest problems facing the electrical engineer engaged in circuit synthesis is that of finding a circuit with a given impedance or transfer function which is better than all others in some subsidiary aspect. There are various criteria that can be, and have been, used for comparison between circuits with identical port characteristics. Examples of these include the sensitivity to component variations, the number or type of components, the overall complexity of the circuits, and several other like measures.

There are two things that must be accomplished, however, before a search can be instituted to find the best circuits. First, there must be a method for finding at least one circuit with the proper port characteristics. This problem has been solved in many forms, for many types of characteristics, and, although there are gaps to be filled, most functions can now be realized. Second, there must be a method for generating a great number of equivalent circuits either from the first circuit or from the characteristics. Because of the rigidity of most synthesis procedures, it appears easier to generate a single circuit and then apply suitable transformations to find a set of equivalent circuits. Once the equivalent circuits are known, the search for the best can be begun.

This thesis deals with two of the most promising of the several equivalence transformations devised over the years by the leaders in the field. The first, by R. E. Kalman [6], is based on the state equations of the circuit and is able to preserve the transfer function from input to output of the system. In the case developed of the one-port passive circuit, the input is taken to be either the port voltage or current, and the output the other characteristic. Thus, the transfer function from input to output is an immitance, and this is maintained by the transformation.

The second transformation, developed by Nathan Howitt as early as 1930 [5], is still more powerful, using the loop impedance matrix of the circuit to maintain nearly any port characteristic of an n-port invariant while generating an infinite number of equivalent circuits. Unfortunately, in some cases a great number of these circuits include negative elements which, although realizable at present, increase the complexity of the circuit greatly. Howitt's Congruence Transformation, however, has a great advantage in that the step from the generated impedance matrix to the new circuit is generally much easier than the step from a set of state and output equations to the circuit.

Kalman's transformation has been discussed by Newcomb, Anderson, and Youla [1], and Howitt's has provided the basis for the theory of continuously equivalent circuits

as presented by Schoeffler [9,10], and Ardalan and Parker[2].

In this thesis the Kalman and Howitt transformations are applied to the canonic forms of Foster [7] and Cauer [4,7]. The results provide a direct transformation from one to the other which has not been available before. Given one form it is difficult to reconstitute the impedance function and then resynthesize to obtain another form. These results indicate how a direct transformation from one realization to another can be achieved. The transformations are applied to specific examples and then generalized using n-dimensional matrix formulations.

II. KALMAN'S TRANSFORMATION

Kalman's transformation was first published in the 1965 Allerton Conference Proceedings [6] and expanded subsequently in 1966 [1] as a solution to the problem of generating equivalent circuits. The authors showed how the transformation could be used to find equivalent circuits from the state equations of a first circuit. This chapter presents a derivation of the transformation and then proceeds through several examples selected to show both the strong and weak points. Included are the application of the transformation to the Foster impedance/admittance general LC forms, the Cauer ladder RC forms, and the Brune and Bott-Duffin realizations for second-order impedance functions. The last example demonstrates its incompleteness.

2.1 GENERAL DERIVATION

As stated above, Kalman's transformation is based on the state-equations approach to circuitry, with the object of maintaining the transfer function from input to output. The general form of the state equations can be written

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \qquad 2.1\text{-}1$$

where $\underline{x}$ is an nX1 column vector of the states, $\underline{u}$ is a pX1 column vector of the inputs, the dot signifies time derivative, and $\underline{A}$ and $\underline{B}$ are nXn and nXp matrices, respectively. The general form of the output equation is as at the top of the next page.

$$\underline{y} = \underline{C}\,\underline{x} + \underline{D}\,\underline{u} \qquad 2.1\text{-}2$$

where $\underline{y}$ is a qX1 column vector of the outputs, and $\underline{C}$ and $\underline{D}$ are qXn and qXp respectively. The pertinent transfer function from $\underline{u}$ to $\underline{y}$ defined by

$$\underline{y} = \underline{W}\,\underline{u} \qquad 2.1\text{-}3$$

where $\underline{W}$ is a qXp matrix, can be expressed as

$$\underline{W} = \underline{C}\left[s\underline{I} - \underline{A}\right]^{-1}\underline{B} + \underline{D} \qquad 2.1\text{-}4$$

by a solution of 2.1-1 and 2.1-2 using the Laplace transform. In equation 2.1-4, s is the Laplace variable signifying a time derivative, $\underline{I}$ is an nXn identity matrix, and superscript $^{-1}$ signifies inversion of the matrix.

If a new system is formed by transforming the matrices $\underline{A}$, $\underline{B}$, $\underline{C}$, and $\underline{D}$ by the nXn square, non-singular matrix $\underline{T}$ using the following relations,

$$\underline{A}' = \underline{T}\,\underline{A}\,\underline{T}^{-1} \qquad 2.1\text{-}5$$

$$\underline{B}' = \underline{T}\,\underline{B} \qquad 2.1\text{-}6$$

$$\underline{C}' = \underline{C}\,\underline{T}^{-1} \qquad 2.1\text{-}7$$

$$\underline{D}' = \underline{D} \qquad 2.1\text{-}8$$

then the new transfer function is given by

$$\underline{W}' = \underline{C}'\left[s\underline{I} - \underline{A}'\right]^{-1}\underline{B}' + \underline{D}'$$

$$= \underline{C}\,\underline{T}^{-1}\left[s\underline{I} - \underline{T}\,\underline{A}\,\underline{T}^{-1}\right]^{-1}\underline{T}\,\underline{B} + \underline{D} \qquad 2.1\text{-}9$$

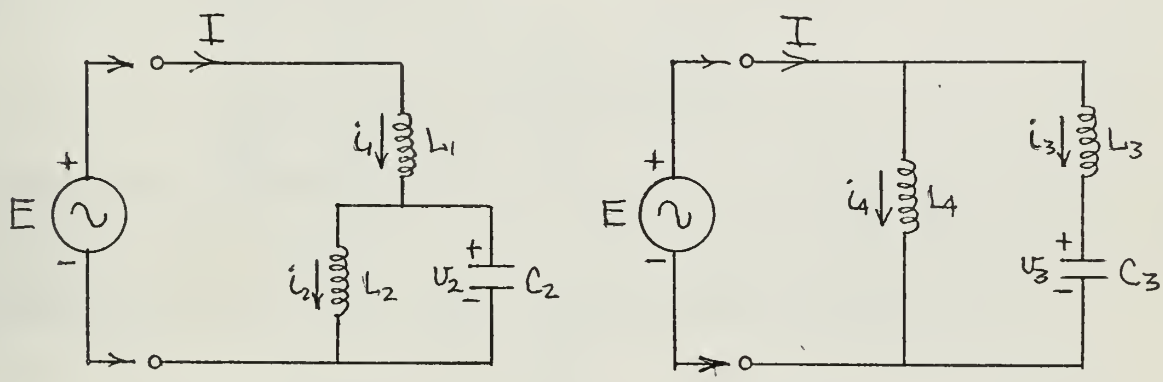
which can be reduced to

$$\underline{W}' = \underline{C}\left[s\underline{I} - \underline{A}\right]^{-1}\underline{B} + \underline{D} = \underline{W} \qquad 2.1\text{-}10$$

Thus, it can be seen that, despite the alterations in $\underline{A}$, $\underline{B}$, and $\underline{C}$, the transfer function from input to output remains

unchanged. The only possible restriction to the generation of equivalent circuits by this method is equation 2.1-8, which requires that the influence of the input directly on the output be the same in all the generated circuits. It will be seen that this can be a definite restriction which limits the set of circuits that can be generated. For the following example, however, both the circuits used have no direct influence between input and output.

2.2 EXAMPLE: Foster Form, Third Order, LC Circuits



Circuit #1   Circuit #2

In order to show how Kalman's transformation can be used to determine equivalence, the two Foster circuits above are used which have the possibility of being equivalent. Both circuits have a zero at zero frequency, a pole at infinity, and a resonance at some intermediate frequency. It will be demonstrated how the transformation matrix $\underline{T}$ is found, and what relations between the circuits' elements are necessary for equivalence.

The first step of the process is to find the state

equations for the two circuits. As stated in the introduction, the port voltage will be used as the input to the system, with the port current the output. Thus, the transfer function W will be the input impedance of the circuits, and the transformation should keep this the same for both circuits.

For circuit #1, the state equations can be written directly in matrix form as

$$\begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} E \qquad 2.2\text{-}1$$

This can be rewritten as

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\Gamma_1 \\ 0 & 0 & \Gamma_2 \\ S_2 & -S_2 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ 0 \\ 0 \end{bmatrix} E \qquad 2.2\text{-}2$$

where, for simplicity, $\Gamma_i = 1/L_i$ and $S_i = 1/C_i$. It should be noted that, for both systems, n=3, p=1, and, in the output equations, q=1. The output equation for circuit #1 is

$$I = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_2 \end{bmatrix} \qquad 2.2\text{-}3$$

For circuit #2, the equations can be as easily written as

$$\begin{bmatrix} L_3 & 0 & 0 \\ 0 & L_4 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \dot{i}_3 \\ \dot{i}_4 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \\ v_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} E \qquad 2.2\text{-}4$$

or

$$\begin{bmatrix} \dot{i}_3 \\ \dot{i}_4 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\Gamma_3 \\ 0 & 0 & 0 \\ S_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \\ v_3 \end{bmatrix} + \begin{bmatrix} \Gamma_3 \\ \Gamma_4 \\ 0 \end{bmatrix} E \qquad 2.2\text{-}5$$

and the output equation is

$$I = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \\ v_3 \end{bmatrix} \qquad 2.2\text{-}6$$

To summarize, then, the matrices needed for the transformation are

$$\underline{A}=\begin{bmatrix} 0 & 0 & -\Gamma_1 \\ 0 & 0 & \Gamma_2 \\ S_2 & -S_2 & 0 \end{bmatrix} \quad \underline{B}=\begin{bmatrix} \Gamma_1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{C}=[1\ 0\ 0] \quad \underline{D}=0$$

$$\underline{A}'=\begin{bmatrix} 0 & 0 & -\Gamma_3 \\ 0 & 0 & 0 \\ S_3 & 0 & 0 \end{bmatrix} \quad \underline{B}'=\begin{bmatrix} \Gamma_3 \\ \Gamma_4 \\ 0 \end{bmatrix} \quad \underline{C}'=[1\ 1\ 0] \quad \underline{D}'=0 \qquad 2.2\text{-}7$$

where the prime refers to the second circuit to distinguish it from the first. It is important to note that, in these matrices, $\underline{D}'=\underline{D}$, a condition which must be satisfied in order to use the transformation. To facilitate the finding of the $\underline{T}$ matrix, the transformation equations 2.1-5 through 2.1-7 are rearranged to remove the inverse:

$$\left.\begin{array}{ll} \underline{A}'\underline{T}=\underline{T}\,\underline{A} & \underline{C}=\underline{C}'\underline{T} \\ \underline{B}'=\underline{T}\,\underline{B} & \end{array}\right\} \quad 2.2\text{-}8$$

From these, $\underline{T}$ may be found directly by assuming a solution and substituting into equations 2.2-8. Proceeding along these lines, the assumed solution for $\underline{T}$ is

$$\underline{T}=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \qquad 2.2\text{-}9$$

Then equations 2.2-8 are used:

$\boxed{\underline{C}=\underline{C}'\underline{T}}$ becomes

$$[1\ \ 0\ \ 0]=[1\ \ 1\ \ 0]\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \quad [(a+d)\ (b+e)\ (c+f)] \qquad 2.2\text{-}10$$

This provides three scalar equations to be used in finding the elements of $\underline{T}$:

$$a+d=1 \qquad b+e=0 \qquad c+f=0 \qquad 2.2\text{-}11$$

$\boxed{B' = TB}$ becomes

$$\begin{bmatrix} \Gamma_3 \\ \Gamma_4 \\ 0 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Gamma_1 a \\ \Gamma_1 d \\ \Gamma_1 g \end{bmatrix} \qquad 2.2\text{-}12$$

Equation 2.2-12 provides another three equations:

$$\Gamma_3 = \Gamma_1 a , \quad \Gamma_4 = \Gamma_1 d , \qquad 0 = \Gamma_1 g \qquad 2.2\text{-}13$$

$\boxed{A'T = TA}$ becomes

$$\begin{bmatrix} 0 & 0 & -\Gamma_3 \\ 0 & 0 & 0 \\ S_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \begin{bmatrix} 0 & 0 & \Gamma_1 \\ 0 & 0 & \Gamma_2 \\ S_2 & -S_2 & 0 \end{bmatrix} \qquad 2.2\text{-}14$$

or

$$\begin{bmatrix} -\Gamma_3 g & -\Gamma_3 h & -\Gamma_3 j \\ 0 & 0 & 0 \\ S_3 a & S_3 b & S_3 c \end{bmatrix} = \begin{bmatrix} S_2 c & -S_2 c & (-\Gamma_1 a + \Gamma_2 b) \\ S_2 f & -S_2 f & (-\Gamma_1 d + \Gamma_2 e) \\ S_2 j & -S_2 j & (-\Gamma_1 g + \Gamma_2 h) \end{bmatrix} \qquad 2.2\text{-}15$$

This equation provides nine more scalar equations. All fifteen of the scalar equations found are written below, with the dependent equations denoted with an asterisk.

2.2-16	$S_2 c = -\Gamma_3 g$	2.2-17	$S_2 f = 0$
2.2-18	$S_2 j = S_3 a$	2.2-19	$-S_2 c = -\Gamma_3 h$
2.2-20*	$-S_2 f = 0$	2.2-21	$-S_2 j = S_3 b$
2.2-22	$-\Gamma_1 a + \Gamma_2 b = -\Gamma_3 j$	2.2-23	$-\Gamma_1 d + \Gamma_2 e = 0$
2.2-24*	$-\Gamma_1 g + \Gamma_2 h = S_3 c$	2.2-25	$a + d = 1$
2.2-26	$b + e = 0$	2.2-27*	$c + f = 0$
2.2-28	$\Gamma_3 = \Gamma_1 a$	2.2-29	$\Gamma_4 = \Gamma_1 d$
		2.2-30	$0 = \Gamma_1 g$

To show the dependence of the indicated equations, a partial solution is necessary. Equations 2.2-17 and 2.2-30 are trivial, with solutions $f = g = 0$ for non-zero

values of the components. Then equation 2.2-16 has the solution $c=0$ and equation 2.2-19 yields $h=0$. Substituting these four values into equations 2.2-20, 24, and 27 produces $0=0$ in all three cases, showing the dependence.

Four equations have been used and three discarded, leaving eight equations to determine the remaining five elements of $\underline{T}$. Equations 2.2-28 and 29 are trivial, with solutions

$$a=\frac{L_1}{L_3}\ , \quad d=\frac{L_1}{L_4} \qquad 2.2\text{-}31$$

Equation 2.2-18 may then be used to obtain

$$j=\frac{C_2}{C_3}a=\frac{C_2 L_1}{C_3 L_3} \qquad 2.2\text{-}32$$

Then equations 2.2-18 and 21 together form

$$S_3 b = S_2 j = -S_3 a \qquad 2.2\text{-}33$$

or

$$b=-a=-\frac{L_1}{L_3} \qquad 2.2\text{-}34$$

Finally, equation 2.2-26 may be rearranged to obtain

$$e=-b=a=\frac{L_1}{L_3} \qquad 2.2\text{-}35$$

There is now a solution for the $\underline{T}$ matrix as

$$T=\begin{bmatrix} \left(\frac{L_1}{L_3}\right) & \left(-\frac{L_1}{L_3}\right) & 0 \\ \left(\frac{L_1}{L_4}\right) & \left(\frac{L_1}{L_3}\right) & 0 \\ 0 & 0 & \left(\frac{L_1 C_2}{L_3 C_3}\right) \end{bmatrix} \qquad 2.2\text{-}36$$

Three of the fifteen equations, however, have not yet been used. Substituting the solutions above into these three equations produces

$$-\Gamma_3-\frac{L_1}{L_2 L_3}=\frac{-L_1 C_2}{L_3^2 C_3} \qquad 2.2\text{-}37$$

$$-\Gamma_4+\frac{L_1}{L_2 L_3}=0 \qquad 2.2\text{-}38$$

$$\frac{L_1}{L_3}+\frac{L_1}{L_4}=1 \qquad 2.2\text{-}39$$

It is now apparent that these last three equations provide

constraints on the circuits' elements which are necessary for the equivalence of the circuits. The three equations above can be solved for the elements of circuit #2 to yield

$$L_3 = \frac{L_1}{L_2}(L_1 + L_2) \qquad 2.2\text{-}40$$

$$L_4 = L_1 + L_2 \qquad 2.2\text{-}41$$

$$S_3 = S_2\left(\frac{L_1}{L_2} + 1\right)^2 \qquad 2.2\text{-}42$$

To show that the two circuits are indeed equivalent with these constraints, the impedance functions will be compared. For circuit #1,

$$Z_1(s) = sL_1 + \frac{\frac{1}{C_2}s}{s^2 + \frac{1}{L_2C_2}} = \frac{s^3L_1 + sS_2\left(\frac{L_1}{L_2}+1\right)}{s^2 + \frac{1}{L_2C_2}} \qquad 2.2\text{-}43$$

For circuit #2,

$$Z_2(s) = \frac{\left(sL_3 + \frac{1}{C_3s}\right)(sL_4)}{sL_3 + \frac{1}{C_3s} + sL_4} = \frac{s^3\frac{L_3L_4}{L_3+L_4} + s\frac{L_4S_3}{L_3+L_4}}{s^2 + \frac{S_3}{L_3+L_4}} \qquad 2.2\text{-}44$$

Then the constraints for L_3, L_4, and S_3 are substituted in $Z_2(s)$ to produce

$$Z_2(s) = \frac{s^3\left[\frac{L_1/L_2(L_1+L_2)(L_1+L_2)}{L_1/L_2(L_1+L_2)+(L_1+L_2)}\right] + s\left[\frac{(L_1+L_2)S_2\left(\frac{1}{L_2}\right)^2(L_1+L_2)^2}{L_1/L_2(L_1+L_2)+(L_1+L_2)}\right]}{s^2 + \left[\frac{S_2\left[\frac{1}{L_2}(L_1+L_2)\right]^2}{L_1/L_2(L_1+L_2)+(L_1+L_2)}\right]}$$

$$= \frac{s^3L_1 + sS_2\left(\frac{L_1}{L_2}+1\right)}{s^2 + \frac{1}{L_2C_2}} = Z_1(s) \qquad 2.2\text{-}45$$

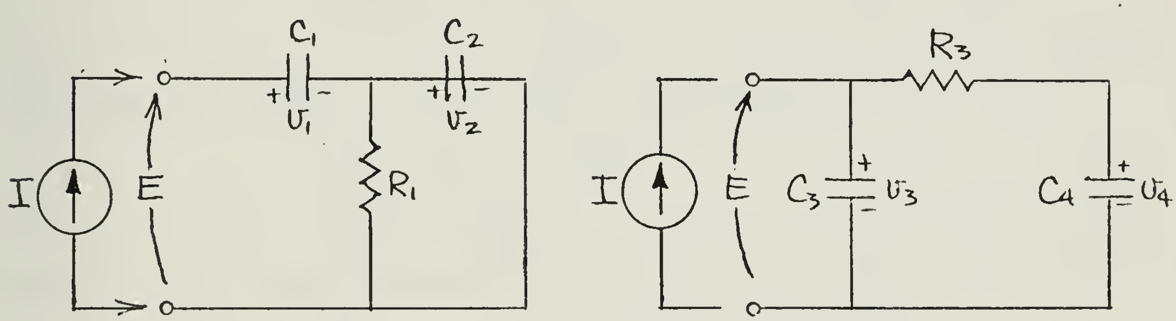
The solution for $\underline{T}$ and the constraints is thus a unique solution, because twelve independent equations have been solved for twelve variables (L_3, L_4, C_3, and the nine elements of $\underline{T}$) in terms of the other three variables (L_1, L_2, and C_2). The matrix T can then be put in terms of the

elements of the first circuit by applying the constraints 2.2-40, 41, and 42:

$$\underline{T} = \begin{bmatrix} \left(\frac{L_2}{L_1+L_2}\right) & \left(\frac{-L_2}{L_1+L_2}\right) & 0 \\ \left(\frac{L_1}{L_1+L_2}\right) & \left(\frac{L_2}{L_1+L_2}\right) & 0 \\ 0 & 0 & \left(\frac{L_1+L_2}{L_2}\right) \end{bmatrix} \qquad 2.2\text{-}46$$

It is interesting to note that the transformation matrix does not depend on the value of the capacitance C_2, even though this capacitance is an integral part of the first circuit and its equations. This stems from the form of equation 2.2-42, the constraint relating the values of the capacitances. The second circuit's capacitance may be obtained from the first's merely by multiplying by a factor determined by the values of the inductors. Thus, the capacitances are included by including this factor, which is disguised in the lower right element of $\underline{T}$.

2.3 EXAMPLE: Cauer Form, Second Order, RC Circuits



Circuit #1 Circuit #2

The two circuits above provide an even easier example in which the matrices are of dimension two rather than three. This example demonstrates that the constraints found

will include those for any resistive elements in the circuits as well as the energy-storage elements. In addition, this is an example of a case where the transfer function $\underline{W}$ is an admittance rather than an impedance. This is brought about by using the port current as the input and the voltage as the output, the opposite of section 2.2.

The state equations for both circuits are written very easily, using the voltages v_1, v_2, v_3, and v_4 as the state variables for the two circuits. For the first circuit, the current equations can be written directly as

$$C_1 \dot{v}_1 = I \qquad 2.3\text{-}1$$

$$C_2 \dot{v}_2 = I - \frac{v_2}{R_1} \qquad 2.3\text{-}2$$

which can be put into matrix form as

$$\begin{bmatrix} \dot{v}_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -G_1 S_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} I \qquad 2.3\text{-}3$$

where $G_i = \frac{1}{R_i}$. The output is simply the sum of the voltages,

$$E = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad 2.3\text{-}4$$

The second circuit has current equations of

$$C_3 \dot{v}_3 = I - \frac{v_3 - v_4}{R_3} \qquad 2.3\text{-}5$$

$$C_4 \dot{v}_4 = \frac{v_3 - v_4}{R_3} \qquad 2.3\text{-}6$$

which again easily form a matrix equation as

$$\begin{bmatrix} \dot{v}_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -G_3 S_3 & G_3 S_3 \\ G_3 S_4 & -G_3 S_4 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} S_3 \\ 0 \end{bmatrix} I \qquad 2.3\text{-}7$$

The output voltage is identical to v_3, so the output equation is

$$E = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} \qquad 2.3\text{-}8$$

The transformation matrix is assumed to be 2X2, or

$$\underline{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad 2.3\text{-}9$$

Then the transformation equations 2.2-8 can be applied to the matrices of equations 2.3-3, 4, 7, and 8:

$\boxed{\underline{C} = \underline{C}'\underline{T}}$ becomes

$$\begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix} \qquad 2.3\text{-}10$$

$\boxed{\underline{B}' = \underline{T}\,\underline{B}}$ becomes

$$\begin{bmatrix} S_3 \\ 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} S_1 a + S_2 b \\ S_1 c + S_2 d \end{bmatrix} \qquad 2.3\text{-}11$$

$\boxed{\underline{A}'\underline{T} = \underline{T}\,\underline{A}}$ becomes

$$\begin{bmatrix} -G_3S_3 & G_3S_3 \\ G_3S_4 & -G_3S_4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & -G_1S_2 \end{bmatrix} \qquad 2.3\text{-}12$$

$$\begin{bmatrix} G_3S_3(c-a) & G_3S_3(d-b) \\ G_3S_4(a-c) & G_3S_4(b-d) \end{bmatrix} = \begin{bmatrix} 0 & -G_1S_2 b \\ 0 & -G_1S_2 d \end{bmatrix} \qquad 2.3\text{-}13$$

Equations 2.3-10, 11, and 13 provide the following eight scalar equations to be solved for the elements of $\underline{T}$ and the components of the second circuit in terms of the components of the first circuit. The one asterisked equation (2.3-19) is dependent, as can be seen by comparison with equation 2.3-18.

2.3-14 $a = 1$ 2.3-15 $b = 1$

2.3-16 $S_3 = S_1 a + S_2 b$ 2.3-17 $0 = S_1 c + S_2 d$

2.3-18 $c - a = 0$ 2.3-19* $a - c = 0$

2.3-20 $G_3S_3(d-b) = -G_1S_2 b$ 2.3-21 $G_3S_4(b-d) = -G_1S_2 d$

The solution to these equations is very simple. Equations 2.3-14 and 15 give the values of a and b, then equation 2.3-18 states that c=a, so c=1. Finally, from equation 2.3-17,

$$d = -\frac{S_1}{S_2}c = -\frac{C_2}{C_1} \qquad 2.3\text{-}22$$

Then the T matrix, in terms of both circuits, is

$$\underline{T} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{C_2}{C_1} \end{bmatrix} \qquad 2.3\text{-}23$$

The three equations remaining provide the constraints, just as in the previous section. By substitution, the following are obtained:

$$S_3 = S_1 + S_2 \qquad 2.3\text{-}24$$

$$G_3 S_3\left(\frac{C_2}{C_1} + 1\right) = G_1 S_2 \qquad 2.3\text{-}25$$

$$G_3 S_4\left(1 + \frac{C_2}{C_1}\right) = G_1 S_1 \qquad 2.3\text{-}26$$

Then the constraints follow through a simultaneous solution of the three equations for S_3, C_4, and R_3:

$$S_3 = S_1 + S_2 \qquad 2.3\text{-}27$$

$$C_4 = \frac{C_1^2}{C_1 + C_2} \qquad 2.3\text{-}28$$

$$R_3 = R_1\left(\frac{C_1 + C_2}{C_1}\right)^2 \qquad 2.3\text{-}29$$

It is not necessary to apply these to T, because the solution in equation 2.3-23 by chance does not include any of the elements of the second circuit.

This section and the preceeding one have shown how the transformation may be used to determine the equivalence of two circuits by solving for the transformation matrix and

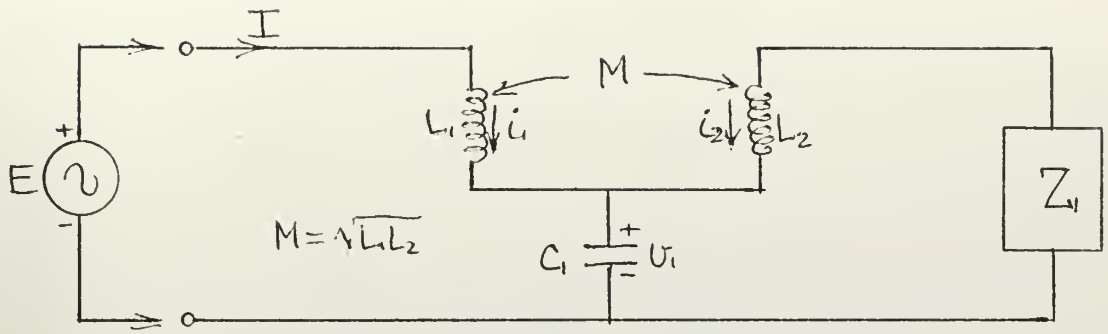
the constraints on the circuit elements. This seems to be a powerful tool which could be used to find the constraints on equivalence of nearly any two possibly equivalent circuits. However, the next section will demonstrate the main deficiency of the Kalman transformation.

2.4 EXAMPLE: Brune and Bott-Duffin Synthesis Circuits

The Brune [3,7,8] and Bott-Duffin [7] circuits are two general forms which can be applied to any RLC impedance function to obtain the initial circuit realization mentioned in the introduction. It would be greatly desirable if a method for transferring quickly from one to the other could be found. At present, the only method to accomplish this is by proceeding through each of the two synthesis procedures, a rather long and involved process.

The following attempt to use Kalman's transformation to find the relations between the circuit elements is found to fail due to the main deficiency of the transformation; namely, the requirement that the circuits used all have the same direct relation between input and output.

The procedure used in the preceeding sections is used here. The general state equations and output equations are found, and then the transformation equations are applied to the matrices of the state and output equations. From this, the result would hopefully yield the transformation matrix and the constraints on the elements of the two circuits.



The Brune Circuit

There is a problem in writing the state equations of the above circuit which lies in the fact that one of the two inductors is excess, despite the fact that it does not lie in a cut-set of inductors. This comes about because of the mutual inductance which places a constraint on one of the inductors. If this constraint is ignored, the apparent state equations can be easily written as

$$\begin{bmatrix} L_1 & \sqrt{L_1L_2} & 0 \\ \sqrt{L_1L_2} & L_2 & 0 \\ 0 & 0 & C_1 \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -Z_1 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} E \qquad 2.4\text{-}1$$

Unfortunately, the premultiplying matrix on the left side of the equation is singular (its determinant equals zero) and therefore has no inverse. Thus, it cannot be inverted and taken to the other side as in the previous examples. This is an indication that there are too many state variables defined.

To reduce the number of state variables, either row operations can be used or, equivalently, the scalar equations can be manipulated. Considering the first two scalar equations of the matrix equation 2.4-1, which can be

removed from the matrices and written

$$L_1 \dot{i}_1 + \sqrt{L_1 L_2}\,\dot{i}_2 = E - v_1 \qquad 2.4\text{-}2$$

$$\sqrt{L_1 L_2}\,\dot{i}_1 + L_2 \dot{i}_2 = -v_1 - i_2 Z_1 \qquad 2.4\text{-}3$$

then equation 2.4-3 can be solved for $\dot{i}_1$:

$$\dot{i}_1 = -\sqrt{\frac{L_2}{L_1}}\,\dot{i}_2 - i_2 \frac{Z_1}{\sqrt{L_1 L_2}} - \frac{v_1}{\sqrt{L_1 L_2}} \qquad 2.4\text{-}4$$

When this is substituted into 2.4-2, equation 2.4-5 is obtained

$$\cancel{-\sqrt{L_1 L_2}\,\dot{i}_2} - i_2 Z_1 \sqrt{\frac{L_1}{L_2}} - v_1 \sqrt{\frac{L_1}{L_2}} + \cancel{\sqrt{L_1 L_2}\,\dot{i}_2} = E - v_1 \qquad 2.4\text{-}5$$

which can be rearranged and solved for i_2:

$$i_2 \sqrt{\frac{L_1}{L_2}}\, Z_1 = v_1\left(1 - \sqrt{\frac{L_1}{L_2}}\right) - E$$

$$i_2 = v_1 \frac{1}{Z_1}\left(\sqrt{\frac{L_2}{L_1}} - 1\right) - E \frac{1}{Z_1}\sqrt{\frac{L_2}{L_1}} \qquad 2.4\text{-}6$$

This can be differentiated with respect to time to get

$$\dot{i}_2 = \dot{v}_1 \frac{1}{Z_1}\left(\sqrt{\frac{L_2}{L_1}} - 1\right) - \dot{E} \frac{1}{Z_1}\sqrt{\frac{L_2}{L_1}} \qquad 2.4\text{-}7$$

Then equations 2.4-6 and 2.4-7 can be substituted into equation 2.4-3 to obtain an expression which does not include i_2 or its derivative:

$$\sqrt{L_1 L_2}\,\dot{i}_1 + \dot{v}_1 \frac{L_2}{Z_1}\left(\sqrt{\frac{L_2}{L_1}} - 1\right) - \dot{E} \frac{L_2}{Z_1}\sqrt{\frac{L_2}{L_1}} = -v_1 - v_1\left(\sqrt{\frac{L_2}{L_1}} - 1\right) + E\sqrt{\frac{L_2}{L_1}} \qquad 2.4\text{-}8$$

This is rearranged to obtain the state equation form

$$\sqrt{L_1 L_2}\,\dot{i}_1 + \frac{L_2}{Z_1}\left(\sqrt{\frac{L_2}{L_1}} - 1\right)\dot{v}_1 = -v_1\sqrt{\frac{L_2}{L_1}} + E\sqrt{\frac{L_2}{L_1}} + \dot{E}\frac{L_2}{Z_1}\sqrt{\frac{L_2}{L_1}} \qquad 2.4\text{-}9$$

Then equation 2.4-6 can be substituted into the third scalar equation of equation 2.4-1 to remove i_2:

$$C_1 \dot{v}_1 = i_1 + v_1 \frac{1}{Z_1}\left(\sqrt{\frac{L_2}{L_1}} - 1\right) - E\frac{1}{Z_1}\sqrt{\frac{L_2}{L_1}} \qquad 2.4\text{-}10$$

At this point, equations 2.4-9 and 2.4-10 will be the

new state equations in only the two states i_1 and v_1. Some work is still necessary to remove $\dot{v}_1$ from the equation 2.4-9. To accomplish this, equation 2.4-10 is solved for $\dot{v}_1$ and then substituted into 2.4-9 to obtain

$$\sqrt{L_1L_2}\,\dot{i}_1 + \left[\frac{L_2}{Z_1}\sqrt{\frac{L_2}{L_1}} - \frac{L_2}{Z_1}\right]\left[\frac{i_1}{C_1} + v_1\frac{1}{Z_1C_1}\left(\sqrt{\frac{L_2}{L_1}}-1\right) - \frac{E}{Z_1C_1}\sqrt{\frac{L_2}{L_1}}\right] = -v_1\sqrt{\frac{L_2}{L_1}} + E\sqrt{\frac{L_2}{L_1}} + \dot{E}\frac{L_2}{Z_1}\sqrt{\frac{L_2}{L_1}} \qquad 2.4\text{-}11$$

Solving algebraically for $\dot{i}_1$, the new state equations become

$$\dot{i}_1 = i_1\left(\frac{\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)}{Z_1C_1L_1}\right) + v_1\left(\frac{2L_2 - \sqrt{L_1L_2} - L_2\sqrt{\frac{L_2}{L_1}} + Z_1^2C_1}{Z_1^2C_1L_1}\right) + E\left(\frac{L_2\left(\sqrt{\frac{L_2}{L_1}}-1\right)+Z_1^2C_1}{Z_1^2C_1L_1}\right) + \dot{E}\frac{L_2}{L_1Z_1} \qquad 2.4\text{-}12$$

$$\dot{v}_1 = \frac{i_1}{C_1} + v_1\left(\frac{\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)}{Z_1C_1L_1}\right) - E\left(\frac{1}{Z_1C_1}\sqrt{\frac{L_2}{L_1}}\right) \qquad 2.4\text{-}13$$

These can be arranged in matrix form by using the Laplace variable and replacing $\dot{E}$ by sE. The matrices are then

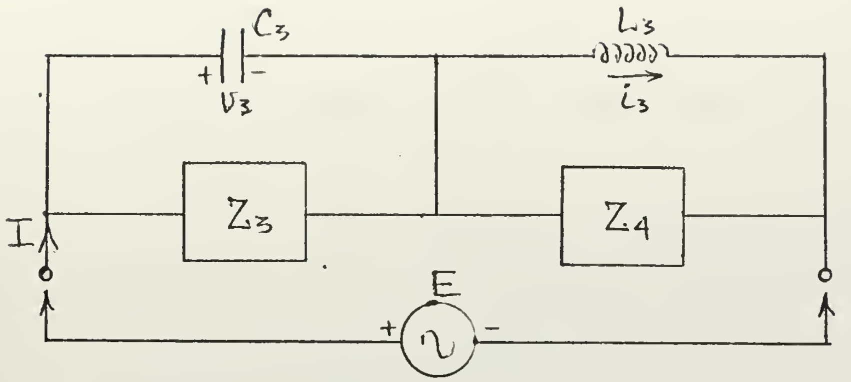
$$\begin{bmatrix}\dot{i}_1\\ \dot{v}_1\end{bmatrix} = \begin{bmatrix}\left(\frac{\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)}{Z_1C_1L_1}\right) & \left(\frac{2L_2-\sqrt{L_1L_2}-L_2\sqrt{\frac{L_2}{L_1}}+Z_1^2C_1}{Z_1^2C_1L_1}\right)\\ \left(\frac{1}{C_1}\right) & \left(\frac{\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)}{Z_1C_1L_1}\right)\end{bmatrix}\begin{bmatrix}i_1\\ v_1\end{bmatrix} + \begin{bmatrix}\left(\frac{L_2\left(\sqrt{\frac{L_2}{L_1}}-1\right)+Z_1C_1(Z_1+sL_2)}{Z_1^2C_1L_1}\right)\\ -\left(\frac{1}{Z_1C_1}\sqrt{\frac{L_2}{L_1}}\right)\end{bmatrix}E \qquad 2.4\text{-}14$$

The output equation is easily written as

$$I = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}i_1\\ v_1\end{bmatrix} \qquad 2.4\text{-}15$$

which has a $\underline{D}$ matrix equal to 0, just as in the previous examples.

Writing the state equations for the Bott-Duffin circuit is a fairly simple matter, unlike the Brune. This circuit, however, (at the top of the next page) presents a problem with respect to the transformation upon the writing of the output equation.



The Bott-Duffin Circuit

Using v_3 and i_3 as the state variables, the state equations can be written on inspection as

$$C_3 \dot{v}_3 = i_3 - \frac{v_3}{Z_3} + \frac{E - v_3}{Z_4} \qquad 2.4\text{-}16$$

$$L_3 \dot{i}_3 = E - v_3 \qquad 2.4\text{-}17$$

The matrix form for these two equations then becomes

$$\begin{bmatrix} \dot{i}_3 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} 0 & -\left(\frac{1}{L_3}\right) \\ \left(\frac{1}{C_3}\right) & \frac{-1}{C_3}\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right) \end{bmatrix} \begin{bmatrix} i_3 \\ v_3 \end{bmatrix} + \begin{bmatrix} \left(\frac{1}{L_3}\right) \\ \left(\frac{1}{C_3 Z_4}\right) \end{bmatrix} E \qquad 2.4\text{-}18$$

Normally, the output equation would be written in the form $\underline{y} = \underline{C}'\ \underline{x}' + \underline{D}'\ \underline{u}$, or

$$I = \begin{bmatrix} 1 & -\frac{1}{Z_4} \end{bmatrix} \begin{bmatrix} i_3 \\ v_3 \end{bmatrix} + \left(\frac{1}{Z_4}\right) E \qquad 2.4\text{-}19$$

In thinking ahead to the transformation, however, it is found that this $\underline{D}'$ does not equal the $\underline{D}$ of the Brune circuit. According to the mathematics of the transformation, this would seem to invalidate the use of Kalman's transformation on these two circuits. It is possible to force $\underline{D}' = 0$ by using the Laplace variable as a differentiator as in the input of the Brune circuit, but the following will show

that this is futile.

By writing the output at the left end of the circuit,

$$I = C_3 \dot{v}_3 + \frac{v_3}{Z_3} \qquad 2.4\text{-}20$$

which converts to

$$I = \begin{bmatrix} 0 & \left(sC_3 + \frac{1}{Z_3}\right) \end{bmatrix} \begin{bmatrix} i_3 \\ v_3 \end{bmatrix} \qquad 2.4\text{-}21$$

and the output equation now has no direct relation between input and output, just as in the Brune case. The transform apparently can now be used, so the procedure of section 2.2 is followed.

The first step is to assume a matrix for $\underline{T}$. In this case, n=2, so the matrix is

$$\underline{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad 2.4\text{-}22$$

The transformation is now applied by using equations 2.2-8:

$\boxed{\underline{C} = \underline{C}'\underline{T}}$ becomes

$$\begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \left(sC_3 + \frac{1}{Z_3}\right) \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c\left(sC_3 + \frac{1}{Z_3}\right) & d\left(sC_3 + \frac{1}{Z_3}\right) \end{bmatrix} \qquad 2.4\text{-}23$$

$\boxed{\underline{B}' = \underline{T}\,\underline{B}}$ becomes

$$\begin{bmatrix} \frac{1}{L_3} \\ \frac{1}{C_3 Z_4} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \left(\frac{L_2\left(\sqrt{\frac{L_2}{L_1}} - 1\right) + Z_1 C_1 (Z_1 + sL_2)}{Z_1^2 C_1 L_1}\right) \\ -\left(\frac{1}{Z_1 C_1}\sqrt{\frac{L_2}{L_1}}\right) \end{bmatrix} = \begin{bmatrix} \left(\frac{aL_2\left(\sqrt{\frac{L_2}{L_1}} - 1\right) + aZ_1 C_1 (Z_1 + sL_2) - bZ_1\sqrt{L_1 L_2}}{Z_1^2 C_1 L_1}\right) \\ \left(\frac{cL_2\left(\sqrt{\frac{L_2}{L_1}} - 1\right) + cZ_1 C_1 (Z_1 + sL_2) - dZ_1\sqrt{L_1 L_2}}{Z_1^2 C_1 L_1}\right) \end{bmatrix} \qquad 2.4\text{-}24$$

$\boxed{\underline{A}'\underline{T} = \underline{T}\,\underline{A}}$ becomes

$$\begin{bmatrix} 0 & -\left(\frac{1}{L_3}\right) \\ \left(\frac{1}{C_3}\right) & \frac{-1}{C_3}\left(\frac{1}{Z_3} + \frac{1}{Z_4}\right) \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \left(\frac{\sqrt{L_1 L_2}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)}{Z_1 C_1 L_1}\right) & \left(\frac{2L_2 - \sqrt{L_1 L_2} - L_2\sqrt{\frac{L_2}{L_1}} + Z_1^2 C_1}{Z_1^2 C_1 L_1}\right) \\ \left(\frac{1}{C_1}\right) & \left(\frac{\sqrt{L_1 L_2}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)}{Z_1 C_1 L_1}\right) \end{bmatrix} \qquad 2.4\text{-}25$$

$$\begin{bmatrix} -\left(\frac{c}{L_3}\right) & -\left(\frac{d}{L_3}\right) \\ \left(\frac{a-c\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right)}{C_3}\right) & \left(\frac{b-d\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right)}{C_3}\right) \end{bmatrix} =$$

$$\begin{bmatrix} \left(\frac{a\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)+bZ_1L_1}{Z_1C_1L_1}\right) & \left(\frac{2aL_2-a\sqrt{L_1L_2}-aL_2\sqrt{\frac{L_2}{L_1}}+aZ_1^2C_1+bZ_1\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)}{Z_1^2C_1L_1}\right) \\ \left(\frac{c\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)+dZ_1L_1}{Z_1C_1L_1}\right) & \left(\frac{2cL_2-c\sqrt{L_1L_2}-cL_2\sqrt{\frac{L_2}{L_1}}+cZ_1^2C_1+dZ_1\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)}{Z_1^2C_1L_1}\right) \end{bmatrix} \quad 2.4\text{-}26$$

Then the three matrix equations 2.4-23, 24, and 26 provide eight scalar equations:

2.4-27 $c\left(sC_3+\frac{1}{Z_3}\right)=1$ 2.4-28 $d\left(sC_3+\frac{1}{Z_3}\right)=0$

$$Z_1^2C_1L_1 = aL_2L_3\left(\sqrt{\frac{L_2}{L_1}}-1\right)+aL_3Z_1^2C_1+asL_3Z_1C_1L_2-bL_3Z_1\sqrt{L_1L_2} \quad 2.4\text{-}29$$

$$Z_1^2C_1L_1 = cL_2C_3Z_4\left(\sqrt{\frac{L_2}{L_1}}-1\right)+cC_3Z_4Z_1^2C_1+csZ_4C_3Z_1C_1L_2-dC_3Z_1Z_4\sqrt{L_1L_2} \quad 2.4\text{-}30$$

$$-cZ_1C_1L_1 = aL_3\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)+bL_1L_3Z_1 \quad 2.4\text{-}31$$

$$aZ_1C_1L_1-cZ_1C_1L_1\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right) = cC_3\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right)+dC_3L_1L_3Z_1 \quad 2.4\text{-}32$$

$$-dZ_1^2C_1L_1 = 2aL_2L_3-aL_3\sqrt{L_1L_2}-aL_2L_3\sqrt{\frac{L_2}{L_1}}+aL_3Z_1^2C_1+bL_3Z_1\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right) \quad 2.4\text{-}33$$

$$bZ_1^2C_1L_1-dZ_1^2C_1L_1\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right) = 2cL_2C_3-cC_3\sqrt{L_1L_2}-cC_3L_2\sqrt{\frac{L_2}{L_1}}+cC_3Z_1^2C_1 + dC_3Z_1\sqrt{L_1L_2}\left(1-\sqrt{\frac{L_2}{L_1}}\right) \quad 2.4\text{-}34$$

These eight equations would be normally solved, in terms of the Brune elements (L_1, L_2, C_1, and Z_1), for the four elements of $\underline{T}$ (a, b, c, and d) and the four elements of the Bott-Duffin circuit (L_3, C_3, Z_3, and Z_4). However, for the purpose of showing that the inclusion of Laplace variables to force $\underline{D}'=0$ is not valid, it will only be necessary to find the elements of $\underline{T}$. Using equations 2.4-27 and 28,

2.4-35 $d=0$ and 2.4-36 $c=\dfrac{1}{sC_3+\frac{1}{Z_3}}$

Then equation 2.4-32 can be used to find a:

$$a Z_1 C_1 L_1 - \frac{Z_1 C_1 L_1 \left(\frac{1}{Z_3} + \frac{1}{Z_4}\right) + C_3 \sqrt{L_1 L_2}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)}{sC_3 + \frac{1}{Z_3}} = 0 \qquad 2.4\text{-}37$$

$$a = \frac{\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{C_3}{Z_1 C_1}\sqrt{\frac{L_2}{L_1}}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)}{sC_3 + \frac{1}{Z_3}} \qquad 2.4\text{-}38$$

Finally, equation 2.4-31 can be used to find b:

$$\frac{-Z_1 C_1 L_1}{sC_3 + \frac{1}{Z_3}} = \frac{\left[\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{C_3}{Z_1 C_1}\sqrt{\frac{L_2}{L_1}}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)\right]\left[L_3 \sqrt{L_1 L_2}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)\right]}{sC_3 + \frac{1}{Z_3}} + b L_1 L_3 Z_1 \qquad 2.4\text{-}39$$

$$b = -\left(\frac{Z_1 C_1 L_1 + L_3 \sqrt{L_1 L_2}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)\left(\frac{1}{Z_3} + \frac{1}{Z_4}\right) + \frac{C_3 L_2 L_3}{Z_1 C_1}\left(1 - \sqrt{\frac{L_2}{L_1}}\right)^2}{Z_1 L_1 L_3 \left(sC_3 + \frac{1}{Z_3}\right)}\right) \qquad 2.4\text{-}40$$

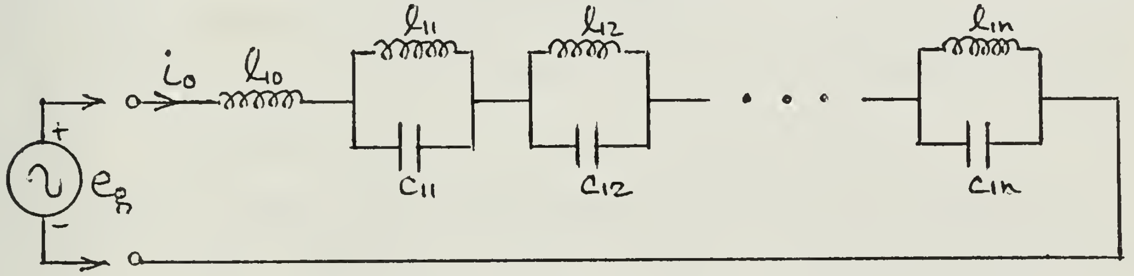
In these solutions for the elements of $\underline{T}$, the Laplace variable is included in all but one of the elements in such a manner that no solution for C_3, Z_3, Z_4, and L_3 could remove it. Thus, the transformation matrix $\underline{T}$ will definitely include s. This implies that, when $\underline{T}$ is applied to $\underline{A}$, the resultant $\underline{A}'$ will also include s, and it is known, by equation 2.4-18, that this is not so. Therefore, the transformation found is inconsistant with the facts, showing that the inclusion of Laplace s to force $\underline{D}' = 0$ is not a valid procedure.

Kalman's transformation is thus not all-inclusive. When the $\underline{D}$ matrices of the two circuits differ, there seems to be no way to use the transformation. This implies that there is a large class of circuits which cannot be found with the transformation.

2.4 APPLICATION TO GENERAL FOSTER FORMS

In some cases, Kalman's transformation may be applied to general n-dimensional forms for the state equations of a type of circuit. Whether or not this can be done depends on the form of the general state equations. In general, when the state equations can be easily partitioned into several standard matrices, then it is possible to apply the transformation.

In the case of the Foster-form circuits, both forms can be partitioned in the same manner to obtain identity matrices and rows and columns of 1's. The circuits used are defined as being of order 2n+1, where n is the number of LC resonances in the circuit. Again, the first step in the procedure is to write state equations for the circuits.



Impedance-form Foster

For the impedance-form Foster circuit above, the first equation involves the voltage across the inductor ,
which can be written

$$l_{10}\dot{i}_{10} = e_g - \sum_{j=1}^{n} v_{1j} \qquad 2.5\text{-}1$$

where the subscripts of the i's and v's correspond to the subscripts of the elements with which they are associated.

The remaining voltage equations are all alike, of the form

$$l_{1j}\,\dot{i}_{1j} = v_{1j} \qquad 2.5\text{-}2$$

The current equations are all of the form

$$c_{1j}\,\dot{i}_{1j} = i_{10} - i_{1j} \qquad 2.5\text{-}3$$

Then equations 2.5-1, 2, and 3 may be joined in partitioned matrix form as

$$\begin{array}{cc} & \begin{array}{ccc} 1 & n & n \end{array} \\ \begin{array}{c} 1 \\ n \\ n \end{array} & \left[\begin{array}{c|c|c} l_{10} & 0 & 0 \\ \hline 0 & L_1 & 0 \\ \hline 0 & 0 & C_1 \end{array}\right] \end{array} \overset{\bullet}{\left[\begin{array}{c} i_{10} \\ \hline I_1 \\ \hline V_1 \end{array}\right]} = \left[\begin{array}{c|c|c} 0 & 0 & -1^{(r)} \\ \hline 0 & 0 & I \\ \hline 1^{(c)} & -I & 0 \end{array}\right] \left[\begin{array}{c} i_{10} \\ \hline I_1 \\ \hline V_1 \end{array}\right] + \left[\begin{array}{c} 1 \\ \hline 0 \\ \hline 0 \end{array}\right] e_g \qquad 2.5\text{-}4$$

where a) lower-case letters indicate scalars, upper-case indicate matrices;

b) $1^{(c)}$ is a column (nX1) of 1's, and $1^{(r)}$ is a row (1Xn) of 1's, but 1 is a scalar;

c) I is an nXn identity matrix;

d) L_1 and C_1 are diagonal matrices of element values, starting with l_{11} and c_{11}, respectively;

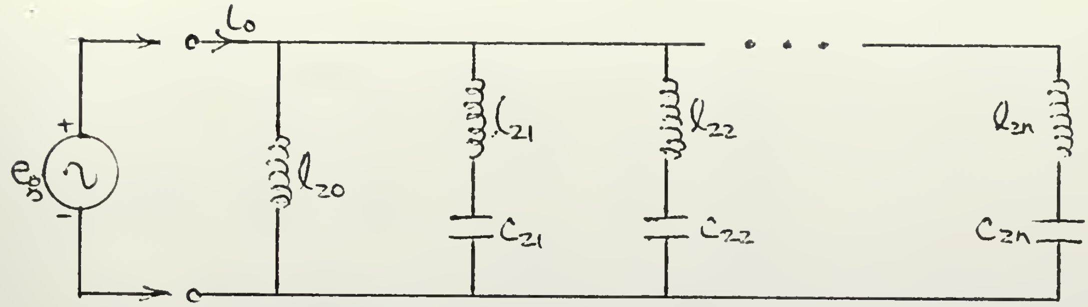
e) I_1 and V_1 are column matrices of the state currents (beginning with i_{11}) and voltages, respectively.

The output equation can be written in the same form as

$$i_0 = \left[\begin{array}{c|c|c} 1 & c & c \end{array}\right] \left[\begin{array}{c} i_{10} \\ \hline I_1 \\ \hline V_1 \end{array}\right] \qquad 2.5\text{-}5$$

where the partitioning is as before.

The same type of reasoning may be applied to the admittance-form Foster circuit on the next page to obtain the state equations and output equation as stated below the circuit, where all the matrices are defined as before.



Admittance-form Foster

$$\begin{array}{cc} & \begin{array}{ccc} 1 & n & n \end{array} \\ \begin{array}{c} 1 \\ n \\ n \end{array} & \left[\begin{array}{c|c|c} l_{20} & 0 & 0 \\ \hline 0 & L_2 & 0 \\ \hline 0 & 0 & C_2 \end{array}\right] \end{array} \left[\begin{array}{c} \dot{i}_{20} \\ \hline \dot{I}_2 \\ \hline \dot{V}_2 \end{array}\right] = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & -I \\ \hline 0 & I & 0 \end{array}\right] \left[\begin{array}{c} i_{20} \\ \hline I_2 \\ \hline V_2 \end{array}\right] + \left[\begin{array}{c} 1 \\ \hline 1^{(c)} \\ \hline 0 \end{array}\right] e_g \qquad 2.5\text{-}6$$

$$i_0 = \left[1 \,\vdots\, 1^{(r)} \,\vdots\, 0\right] \left[\begin{array}{c} i_{20} \\ \hline I_2 \\ \hline V_2 \end{array}\right] \qquad 2.5\text{-}7$$

Then the matrices to be used in Kalman's transformation are

$$\underline{A} = \begin{bmatrix} 0 & 0 & -\gamma_{10}^{(r)} \\ 0 & 0 & \Gamma_1 \\ S_1^{(c)} & -S_1 & 0 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} \gamma_{10} \\ 0 \\ 0 \end{bmatrix} \quad \underline{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad 2.5\text{-}8$$

$$\underline{A}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\Gamma_2 \\ 0 & S_2 & 0 \end{bmatrix} \quad \underline{B}' = \begin{bmatrix} \gamma_{20} \\ \Gamma_2^{(c)} \\ 0 \end{bmatrix} \quad \underline{C}' = \begin{bmatrix} 1 & 1^{(r)} & 0 \end{bmatrix} \qquad 2.5\text{-}9$$

where, for convenience, $\gamma_j = \frac{1}{l_j}$ and $s_j = \frac{1}{c_j}$. The notation to be used throughout this section in referring to row and column matrices is as follows: superscript $^{(c)}$ denotes a column matrix, superscript $^{(r)}$ denotes a row matrix; if the element with one of these superscripts is a scalar, it is repeated throughout the row or column; if the element is a diagonal matrix, the diagonal elements are in order in the row or column. Thus, the matrices used in equations

2.5-8 and 2.5-9 above are

$$\gamma_{10}^{(r)}=\begin{bmatrix}\gamma_{10} & \gamma_{10} & \cdots & \gamma_{10}\end{bmatrix}\ ; \quad S_1^{(c)}=\begin{bmatrix}s_{11}\\ s_{12}\\ \vdots \\ s_{1n}\end{bmatrix}\ ; \quad \Gamma_2^{(c)}=\begin{bmatrix}\gamma_{21}\\ \gamma_{22}\\ \vdots \\ \gamma_{2n}\end{bmatrix}$$

The transformation matrix is assumed to be partitioned in the same manner:

$$\underline{T}=\begin{array}{c} \\ 1 \\ n \\ n \end{array}\begin{array}{c} \begin{array}{ccc} 1 & n & n \end{array} \\ \left[\begin{array}{c|c|c} t_{11} & T_{12} & T_{13} \\ \hline T_{21} & T_{22} & T_{23} \\ \hline T_{31} & T_{32} & T_{33} \end{array}\right] \end{array} \qquad 2.5\text{-}10$$

The transformation equations 2.2-8 are then used as before with the partitioned matrices of equations 2.5-8, 9, and 10:

$\boxed{\underline{C}=\underline{C}'\underline{T}}$ becomes

$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}=\begin{bmatrix}1 & 1^{(r)} & 0\end{bmatrix}\begin{bmatrix}t_{11} & T_{12} & T_{13}\\ T_{21} & T_{22} & T_{23}\\ T_{31} & T_{32} & T_{33}\end{bmatrix}=\begin{bmatrix}(t_{11}+1^{(r)}t_{21}) & (T_{12}+1^{(r)}T_{22}) & (T_{13}+1^{(r)}T_{23})\end{bmatrix} \qquad 2.5\text{-}11$$

$\boxed{\underline{B}'=\underline{T}\,\underline{B}}$ becomes

$$\begin{bmatrix}\gamma_{20}\\ \Gamma_2^{(c)}\\ 0\end{bmatrix}=\begin{bmatrix}t_{11} & T_{12} & T_{13}\\ T_{21} & T_{22} & T_{23}\\ T_{31} & T_{32} & T_{33}\end{bmatrix}\begin{bmatrix}\gamma_{10}\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}t_{11}\gamma_{10}\\ T_{21}\gamma_{10}\\ T_{31}\gamma_{10}\end{bmatrix} \qquad 2.5\text{-}12$$

$\boxed{\underline{\Lambda}\,\underline{T}=\underline{T}\,\underline{A}}$ becomes

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -\Gamma_2\\ 0 & S_2 & 0\end{bmatrix}\begin{bmatrix}t_{11} & T_{12} & T_{13}\\ T_{21} & T_{22} & T_{23}\\ T_{31} & T_{32} & T_{33}\end{bmatrix}=\begin{bmatrix}t_{11} & T_{12} & T_{13}\\ T_{21} & T_{22} & T_{23}\\ T_{31} & T_{32} & T_{33}\end{bmatrix}\begin{bmatrix}0 & 0 & -\gamma_{10}^{(r)}\\ 0 & 0 & \Gamma_1\\ S_1^{(c)} & -S_1 & 0\end{bmatrix} \qquad 2.5\text{-}13$$

or

$$\begin{bmatrix}0 & 0 & 0\\ -\Gamma_2T_{31} & -\Gamma_2T_{32} & -\Gamma_2T_{33}\\ +S_2T_{21} & S_2T_{22} & S_2T_{23}\end{bmatrix}=\begin{bmatrix}T_{13}S_1^{(c)} & -T_{13}S_1 & (T_{12}\Gamma_1-t_{11}\gamma_{10}^{(r)})\\ T_{23}S_1^{(c)} & -T_{23}S_1 & (T_{22}\Gamma_1-T_{21}\gamma_{10}^{(r)})\\ T_{33}S_1^{(c)} & -T_{33}S_1 & (T_{32}\Gamma_1-T_{31}\gamma_{10}^{(r)})\end{bmatrix} \qquad 2.5\text{-}14$$

Then equations 2.5-11, 12, and 14 provide fifteen submatrix

equations of varying dimension which are to be solved:

2.5-15 $t_{11} + 1^{(r)}T_{21} = 1$

2.5-16 $T_{12} + 1^{(r)}T_{22} = 0$

2.5-17* $T_{13} + 1^{(r)}T_{23} = 0$

2.5-18 $\gamma_{20} = t_{11}\gamma_{10}$

2.5-19 $\Gamma_2^{(c)} = T_{21}\gamma_{10}$

2.5-20 $0 = T_{31}\gamma_{10}$

2.5-21* $0 = T_{13}S_1^{(c)}$

2.5-22 $0 = -T_{13}S_1$

2.5-23 $0 = T_{12}\Gamma_1 - t_{11}\gamma_{10}^{(r)}$

2.5-24 $-\Gamma_2 T_{31} = T_{23}S_1^{(c)}$

2.5-25 $-\Gamma_2 T_{32} = -T_{23}S_1$

2.5-26 $-\Gamma_2 T_{33} = T_{22}\Gamma_1 - T_{21}\gamma_{10}^{(r)}$

2.5-27 $S_2 T_{21} = T_{33}S_1^{(c)}$

2.5-28 $S_2 T_{22} = -T_{33}S_1$

2.5-29* $S_2 T_{23} = T_{32}\Gamma_1 - T_{31}\gamma_{10}^{(r)}$

As before, the asterisked equations are dependent. This will be shown in the course of solution.

Because S_1 is a diagonal matrix, there is no adding of terms in equation 2.5-22, and the solution is

$$T_{13} = 0 \qquad 2.5\text{-}30$$

It can be seen immediately that equation 2.5-21 is dependent. From equation 2.5-18,

$$t_{11} = \frac{\gamma_{20}}{\gamma_{10}} = \frac{\ell_{10}}{\ell_{20}} \qquad 2.5\text{-}31$$

Equation 2.5-19 produces

$$T_{21} = \frac{1}{\gamma_{10}}\Gamma_2^{(c)} = \ell_{10}\Gamma_2^{(c)} \qquad 2.5\text{-}32$$

From equation 2.5-20

$$T_{31} = 0 \qquad 2.5\text{-}33$$

and equation 2.5-23, $t_{11}\gamma_{10}^{(r)} = T_{12}\Gamma_1$

$$T_{12} = t_{11}\gamma_{10}^{(r)}L_1 = t_{11}\gamma_{10}L_1^{(r)} = \gamma_{20}L_1^{(r)} \qquad 2.5\text{-}34$$

Then equation 2.5-24 has

$$0 = T_{23}S_1^{(c)} \qquad T_{23} = 0 \qquad 2.5\text{-}35$$

because the equation must equal 0, no matter what the values of the capacitances. It then follows that equations

2.5-17 and 29 are dependent. Finally, from equation 2.5-25,

$$-\Gamma_2 T_{32} = 0 \qquad \text{and} \qquad T_{32} = 0 \qquad 2.5\text{-}36$$

At this point, only T_{22} and T_{33} remain to be found. It is necessary, however, to delve inside of the submatrices to solve for the form of their elements. By defining

$$T_{33} = \begin{bmatrix} t_{11}^{33} & t_{12}^{33} & \cdots & t_{1n}^{33} \\ t_{21}^{33} & t_{22}^{33} & & \vdots \\ \vdots & & & \vdots \\ t_{n1}^{33} & \cdots & \cdots & t_{nn}^{33} \end{bmatrix} \qquad 2.5\text{-}37$$

then equation 2.5-28 can be used to obtain

$$T_{22} = -C_2 T_{33} S_1 = -\begin{bmatrix} c_{21} & 0 & \cdots & 0 \\ 0 & c_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & c_{2n} \end{bmatrix} \begin{bmatrix} t_{11}^{33} & \cdots & t_{1n}^{33} \\ \vdots & & \vdots \\ t_{n1}^{33} & \cdots & t_{nn}^{33} \end{bmatrix} \begin{bmatrix} \Delta_{11} & 0 & \cdots & 0 \\ 0 & \Delta_{12} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \Delta_{1n} \end{bmatrix}$$

$$= -\begin{bmatrix} t_{11}^{33} c_{21} \Delta_{11} & t_{12}^{33} c_{21} \Delta_{12} & \cdots & t_{1n}^{33} c_{21} \Delta_{1n} \\ t_{21}^{33} c_{22} \Delta_{11} & t_{22}^{33} c_{22} \Delta_{12} & & \vdots \\ \vdots & & & \vdots \\ t_{n1}^{33} c_{2n} \Delta_{11} & \cdots & \cdots & t_{nn}^{33} c_{2n} \Delta_{1n} \end{bmatrix} \qquad 2.5\text{-}38$$

This result can then be substituted into equation 2.5-26 in the form

$$T_{21} \gamma_{10}^{(r)} = \Gamma_2 T_{33} + T_{22} \Gamma_1 \qquad 2.5\text{-}39$$

$$\begin{bmatrix} i_{10}\gamma_{21} \\ i_{10}\gamma_{22} \\ \vdots \\ i_{10}\gamma_{2n} \end{bmatrix} \begin{bmatrix} \gamma_{10} & \gamma_{10} & \cdots & \gamma_{10} \end{bmatrix} = \begin{bmatrix} \gamma_{21} & 0 & \cdots & 0 \\ 0 & \gamma_{22} & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & \gamma_{2n} \end{bmatrix} \begin{bmatrix} t_{11}^{33} & \cdots & t_{1n}^{33} \\ \vdots & & \vdots \\ t_{n1}^{33} & \cdots & t_{nn}^{33} \end{bmatrix}$$

$$-\begin{bmatrix} t_{11}^{33} c_{21} \Delta_{11} & t_{12}^{33} c_{21} \Delta_{12} & \cdots & t_{1n}^{33} c_{21} \Delta_{1n} \\ t_{21}^{33} c_{22} \Delta_{11} & t_{22}^{33} c_{22} \Delta_{12} & & \vdots \\ \vdots & & & \vdots \\ t_{n1}^{33} c_{2n} \Delta_{11} & \cdots & \cdots & t_{nn}^{33} c_{2n} \Delta_{1n} \end{bmatrix} \begin{bmatrix} \gamma_{11} & 0 & \cdots & 0 \\ 0 & \gamma_{12} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \gamma_{1n} \end{bmatrix} \qquad 2.5\text{-}40$$

$$\begin{bmatrix} \gamma_{21} & \gamma_{21} & \cdots & \gamma_{21} \\ \gamma_{22} & \gamma_{22} & & \vdots \\ \vdots & & & \vdots \\ \gamma_{2n} & \cdots & \cdots & \gamma_{2n} \end{bmatrix} = \begin{bmatrix} t^{33}_{11}\gamma_{21} & \cdots & t^{33}_{1n}\gamma_{21} \\ \vdots & & \vdots \\ t^{33}_{n1}\gamma_{2n} & \cdots & t^{33}_{nn}\gamma_{2n} \end{bmatrix}$$

$$- \begin{bmatrix} t^{33}_{11}c_{21}\Delta_{11}\gamma_{11} & t^{33}_{12}c_{21}\Delta_{12}\gamma_{12} & \cdots & t^{33}_{1n}c_{21}\Delta_{1n}\gamma_{1n} \\ t^{33}_{21}c_{22}\Delta_{11}\gamma_{11} & t^{33}_{22}c_{22}\Delta_{12}\gamma_{12} & & \vdots \\ \vdots & & & \vdots \\ t^{33}_{n1}c_{2n}\Delta_{11}\gamma_{11} & \cdots & \cdots & t^{33}_{nn}c_{2n}\Delta_{1n}\gamma_{1n} \end{bmatrix} \qquad 2.5\text{-}41$$

Careful examination of equation 2.5-41 reveals that each equality in the equation contains only one t^{33}, and a general form of solution can be written

$$t^{33}_{ij} = \frac{\gamma_{2i}}{\gamma_{2i} - c_{2i}\Delta_{ij}\gamma_{1j}} = \frac{c_{1j}l_{1j}}{c_{1j}l_{1j} - c_{2i}l_{2i}} \qquad 2.5\text{-}42$$

Then equation 2.5-38 can be studied to obtain

$$t^{22}_{ij} = \frac{c_{2i}\Delta_{ij}c_{1j}l_{1j}}{c_{2i}l_{2i} - c_{1j}l_{1j}} = \frac{c_{2i}l_{1j}}{c_{2i}l_{2i} - c_{1j}l_{1j}} \qquad 2.5\text{-}43$$

Then the form of the transformation matrix, in terms of elements of both circuits, is

$$\underline{T} = \begin{array}{c} \\ 1 \\ n \\ n \end{array} \begin{array}{c} \begin{array}{ccc} 1 & n & n \end{array} \\ \left[\begin{array}{c|c|c} \left(\frac{l_{10}}{l_{20}}\right) & \gamma_{20}L_1^{(r)} & 0 \\ \hline l_{10}\Gamma_2^{(c)} & T_{22} & 0 \\ \hline 0 & 0 & T_{33} \end{array}\right] \end{array} \qquad 2.5\text{-}44$$

where the elements of T_{22} and T_{33} are as in equations 2.5-43 and 2.5-42, respectively.

The next step in the procedure is to find the constraints between the elements of the two circuits in order to express $\underline{T}$ in terms of one circuit's elements only, but, as shown below, the equations to be used for this purpose are not solvable in general terms. The three equations not yet used

are 2.5-15, 16, and 27, which are used as follows:

The results given in equation 2.5-44 are substituted into equation 2.5-15 to produce

$$\frac{l_{10}}{l_{20}} + 1^{(r)} l_{10} \Gamma_2^{(c)} = 1 \qquad 2.5\text{-}45$$

which, when the matrix multiplication is performed, may be written as

$$\frac{l_{10}}{l_{20}} + l_{10} \sum_{i=1}^{n} \gamma_{2i} = 1 \qquad 2.5\text{-}46$$

or

$$\sum_{i=0}^{n} \frac{l_{10}}{l_{2i}} = 1 \qquad 2.5\text{-}47$$

The same procedure for equation 2.5-16 produces

$$\gamma_{20} L_1^{(r)} + 1^{(r)} T_{22} = 0 \qquad 2.5\text{-}48$$

or

$$\left[\frac{l_{11}}{l_{20}} \; \frac{l_{12}}{l_{20}} \cdots \frac{l_{1n}}{l_{20}}\right]$$

$$+ \left[\left(\sum_{j=1}^{n} \frac{c_{21} l_{1j}}{c_{21} l_{21} - c_j l_{1j}}\right)\left(\sum_{j=1}^{n} \frac{c_{22} l_{1j}}{c_{22} l_{22} - c_j l_{1j}}\right) \cdots \left(\sum_{j=1}^{n} \frac{c_{2n} l_{1j}}{c_{2n} l_{2n} - c_j l_{1j}}\right)\right] = 0 \qquad 2.5\text{-}49$$

This provides n scalar equations of the form

$$\frac{l_{1k}}{l_{20}} + \sum_{j=1}^{n} \frac{c_{2k} l_{1j}}{c_{2k} l_{2k} - c_j l_{1j}} = 0 \,, \quad k = 1 \text{ to } n \qquad 2.5\text{-}50$$

Lastly, substitution in equation 2.5-27 produces

$$S_2 l_{10} \Gamma_2^{(c)} = T_{33} S_1^{(c)} \qquad 2.5\text{-}51$$

which is, in expanded form,

$$l_{10} \begin{bmatrix} \Delta_{21} & 0 & \cdots & 0 \\ 0 & \Delta_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \Delta_{2n} \end{bmatrix} \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \\ \vdots \\ \gamma_{2n} \end{bmatrix} = T_{33} \begin{bmatrix} \Delta_{11} \\ \Delta_{12} \\ \vdots \\ \Delta_{1n} \end{bmatrix} \qquad 2.5\text{-}52$$

The indicated multiplications produce

$$\begin{bmatrix} l_{10}\Delta_{21}\gamma_{21} \\ l_{10}\Delta_{22}\gamma_{22} \\ \cdot \\ \vdots \\ \cdot \\ l_{10}\Delta_{2n}\gamma_{2n} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\frac{\Delta_{ii}c_{11}l_{11}}{c_{11}l_{11}-c_{2i}l_{2i}} \\ \sum_{i=1}^{n}\frac{\Delta_{ii}c_{12}l_{12}}{c_{12}l_{12}-c_{2i}l_{2i}} \\ \vdots \\ \cdot \\ \sum_{i=1}^{n}\frac{\Delta_{ii}c_{1n}l_{1n}}{c_{1n}l_{1n}-c_{2i}l_{2i}} \end{bmatrix} \qquad 2.5\text{-}53$$

This provides another n equations, of the form

$$l_{10}\Delta_{2k}\gamma_{2k} = \sum_{i=1}^{n}\frac{\Delta_{ii}c_{1k}l_{1k}}{c_{1k}l_{1k}-c_{2i}l_{2i}}, \quad k=1 \text{ to } n \qquad 2.5\text{-}54$$

Unfortunately, although equations 2.5-47, 50, and 54 form 2n+1 equations to find the 2n+1 elements of the admittance-form Foster circuit, equations 2.5-50 and 54 are each of degree n in both c_{2i} and l_{2i}. This prohibits a further general solution, and in addition makes a specific solution very difficult for n>2. Consequently, the transformation matrix must be left in the form of equations 2.5-44, 42, and 43.

2.6 GENERAL OBSERVATIONS

At first glance, Kalman's transformation appears to be a very powerful tool to be used in circuit synthesis, but one soon finds several faults with it. First, and most glaring, is that it requires the direct relation between input and output to be the same for all circuits of a group with which it is used. As demonstrated by section 2.4, this is an unfair requirement to make when a representative sample of circuits with a given characteristic is desired. Circuits with a different D matrix are not even considered, yet the optimum circuit for the engineer's requirements may

very easily lie within the excluded group.

Second, the transformation is based on the state equations, which can be very difficult to write, as in the case of the Brune circuit of section 2.4. In addition, it is almost always difficult to obtain the proper circuit for a certain set of state equations, making the procedure of finding a second circuit from the first synthesized circuit a very difficult one.

Despite these deficits, however, Kalman's transformation is a good tool to be used in limited cases for which it is applicable. It can sometimes be used to find solutions for general classes of circuits which may be transformed back and forth, as in section 2.5. Also, it is a valuable aid in determining the relations between the various elements in two like circuits, as in the Foster LC of section 2.2 and the Cauer RC of section 2.3. For cases of order three or less, it is fairly easy to proceed through the algebra of the transformation to obtain meaningful results, but an increase in the order of the systems greatly increases the complexity of the calculations.

III. HOWITT'S TRANSFORMATION

Howitt's Congruence Transformation was developed in 1930 and first published in The Physical Review in 1931 [5]. Amazingly enough, it is still one of the most powerful transformations available in circuit theory today. Based on the loop impedance function of the circuit, the transformation produces an infinite number of circuits which are topologically congruent with the original, and it can maintain any desired impedance or transfer function of an n-port network. For the purposes of comparison with Kalman's transformation, this thesis will concentrate on the maintenance of the input impedance of a one-port.

3.1 GENERAL DERIVATION

Any RLC circuit can be defined in terms of the loop currents and voltages by the equation

$$\underline{v}_\ell = \underline{Z}\, \underline{i}_\ell \qquad 3.1\text{-}1$$

where $\underline{Z}$ is a square matrix containing the values of the components in the circuit. The $\underline{Z}$ matrix can be split into three matrices, $\underline{R}$, $\underline{L}$, and $\underline{S}$, such that the component values of each type are included in the proper matrix as follows:

a) The main diagonal terms consist of the sum of the particular type of elements around the respective loops.

b) The off-diagonal terms are the values of the elements common to the two loops referenced by the row and column position.

Howitt shows how, by assuming a transformation for the loop currents of

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & & a_{2n} \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} i_1' \\ i_2' \\ \vdots \\ i_n' \end{bmatrix} \qquad 3.1\text{-}2$$

and for the loop voltages of

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & & a_{2n} \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \\ \vdots \\ v_n' \end{bmatrix} \qquad 3.1\text{-}3$$

for an n-mesh circuit, the input impedance to mesh one is maintained constant. A more useful approach is the equivalent procedure of transforming the $\underline{R}$, $\underline{L}$, and $\underline{S}$ matrices by the same $\underline{A}$ matrix and its transpose as

$$\underline{R}' = \underline{A}^t \underline{R} \underline{A} \qquad 3.1\text{-}4$$

$$\underline{L}' = \underline{A}^t \underline{L} \underline{A} \qquad 3.1\text{-}5$$

$$\underline{S}' = \underline{A}^t \underline{S} \underline{A} \qquad 3.1\text{-}6$$

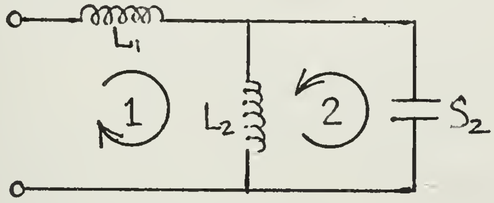
Then these new $\underline{R}'$, $\underline{L}'$, and $\underline{S}'$ define a circuit which will have the $\underline{i}'$ and $\underline{v}'$ of equations 3.1-2 and 3 above.

In order to maintain a transfer function between ports, it is only necessary to have two rows of the $\underline{A}$ matrix filled with a single 1 and zeros. This will not only maintain the input impedances to the two ports designated, but also it will keep the transfer function between the ports constant.

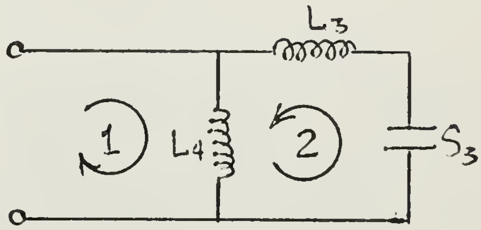
3.2 EXAMPLE: Foster Form, Third Order, LC Circuits

The same circuits used in section 2.2 can be used here to demonstrate Howitt's transformation, and the comparative ease with which it can be used. In this case, the state

equations were of dimension three, and there were thus nine elements of the transformation matrix to be found, but the same circuit has only two loops and is thus defined by a two-dimensional set of loop equations. Because the first row of $\underline{A}$ is already defined, there remain only two elements of the transformation matrix which will have to be found.



Circuit #1



Circuit #2

The circuits are redrawn above to show the distinct loops which will be used in this transformation. The first step is to determine the $\underline{L}$ and $\underline{S}$ matrices for each circuit. (In both cases, $\underline{R}=0$.) In circuit #1, the sum of the second loop's inductances is simply L_2, which is the 2,2 term of the $\underline{L}$ matrix. The sum of the first loop's inductances is L_1+L_2, and this term goes in the 1,1 position on the main diagonal. The off-diagonal terms are equal to the inductance common to both loops, L_2. The same procedure is followed for the capacitance, and the first circuit's matrices are

$$\underline{L}=\begin{bmatrix}(L_1+L_2) & L_2 \\ L_2 & L_2\end{bmatrix} \qquad \underline{S}=\begin{bmatrix}0 & 0 \\ 0 & S_2\end{bmatrix} \qquad 3.2\text{-}1$$

The second circuit's matrices are found in the same way:

$$\underline{L}'=\begin{bmatrix}L_4 & L_4 \\ L_4 & (L_3+L_4)\end{bmatrix} \qquad \underline{S}'=\begin{bmatrix}0 & 0 \\ 0 & S_3\end{bmatrix} \qquad 3.2\text{-}2$$

It is desired to find what $\underline{A}$ matrix will convert circuit #1 into circuit #2. As in Kalman's transformation, a solution is assumed

$$\underline{A} = \begin{bmatrix} 1 & 0 \\ a_{21} & a_{22} \end{bmatrix} \qquad 3.2\text{-}3$$

and substituted into the transformation equations 3.1-5 and 3.1-6 to obtain

$\boxed{\underline{L}' = \underline{A}^t \underline{L} \underline{A}}$ becomes

$$\begin{bmatrix} L_4 & L_4 \\ L_4 & (L_3+L_4) \end{bmatrix} = \begin{bmatrix} (L_1+L_2+2a_{21}L_2+a_{21}^2L_2) & (a_{22}L_2+a_{21}a_{22}L_2) \\ (a_{22}L_2+a_{21}a_{22}L_2) & a_{22}^2L_2 \end{bmatrix} \qquad 3.2\text{-}4$$

$\boxed{\underline{S}' = \underline{A}^t \underline{S} \underline{A}}$ becomes

$$\begin{bmatrix} 0 & 0 \\ 0 & S_3 \end{bmatrix} = \begin{bmatrix} a_{21}^2 S_2 & a_{21}a_{22}S_2 \\ a_{21}a_{22}S_2 & a_{22}^2 S_2 \end{bmatrix} \qquad 3.2\text{-}5$$

Each matrix, because it is symmetric, provides $\frac{1}{2}(n^2+n)$ scalar equations. In this case, n equals 2, and there are six different equations. Of these, however, one is dependent (starred), and there are thus five equations with which to find a_{21}, a_{22}, L_3, L_4, and C_3. The six equations are

3.2-6 $L_4 = L_1+L_2+2a_{21}L_2+a_{21}^2L_2$ 3.2-7 $L_4 = a_{22}L_2+a_{21}a_{22}L_2$

3.2-8 $L_3+L_4 = a_{22}^2L_2$ 3.2-9 $0 = a_{21}^2S_2$

3.2-10* $0 = a_{21}a_{22}S_2$ 3.2-11 $S_3 = a_{22}^2S_2$

For the sake of simplicity, only a_{21} and a_{22} will be found, at which point the constraints of section 2.2 will be used to form $\underline{A}$ in terms of the first circuit alone. It should be understood, however, that these constraints can be found from the above equations, and normally would be.

If it is assumed that the elements are all finite and

non-zero, then equation 3.2-9 produces

$$a_{21}^2 = 0 \qquad \text{or} \qquad a_{21} = 0 \tag{3.2-12}$$

From equation 3.2-11,

$$a_{22} = \sqrt{\frac{S_3}{S_2}} \tag{3.2-13}$$

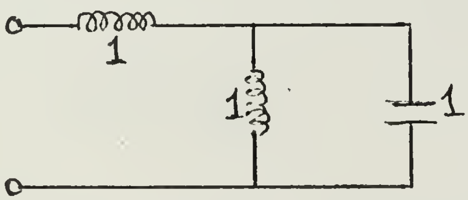
Finally, using equation 2.2-42,

$$a_{22} = \sqrt{\frac{S_2\left(\frac{L_1}{L_2}+1\right)^2}{S_2}} = \frac{L_1}{L_2} + 1 \tag{3.2-14}$$

Then the $\underline{A}$ matrix, in terms of circuit #1, is

$$\underline{A} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{L_1+L_2}{L_2} \end{bmatrix} \tag{3.2-15}$$

One of the best points of Howitt's transformation is that it is relatively easy to determine the circuit from the new $\underline{R}'$, $\underline{L}'$, and $\underline{S}'$, whereas the Kalman transformation ends with the state equations, and it can be a viciously hard step from these equations to the circuit. A good example of this is the use of the above derivation with the first Foster circuit below.



The $\underline{L}$ and $\underline{S}$ matrices of this circuit are

$$\underline{L} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad \underline{S} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.2-16}$$

Using equation 3.2-15 above, the $\underline{A}$ matrix is

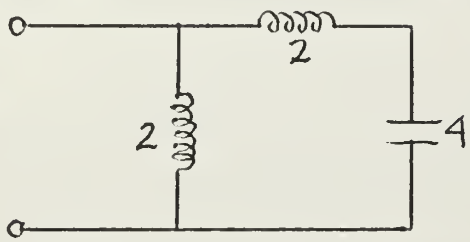
$$\underline{A} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \tag{3.2-17}$$

Applying this to $\underline{L}$ and $\underline{S}$,

$$\underline{L}' = \underline{A}^t \underline{L} \underline{A} = \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix} \tag{3.2-18}$$

$$\underline{S}' = \underline{A}^t \underline{S} \underline{A} = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \qquad 3.2\text{-}19$$

Simply by inspection, these values can be put into a two-mesh circuit to obtain the second Foster circuit



Both of these circuits have the same input impedance

$$Z(s) = \frac{s^3 + 2s}{s^2 + 1} \qquad 3.2\text{-}20$$

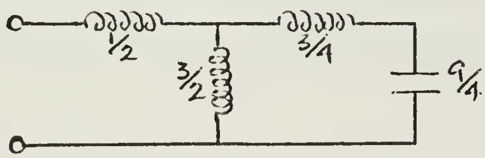
which has been maintained by the transformation. It is interesting to note, however, that this is only one $\underline{A}$ matrix of an infinity which, when applied to the first circuit, produces a realizable second circuit. The second circuit will have two loops, as above, but elements will have different values. For example, applying the $\underline{A}$ matrix

$$\underline{A} = \begin{bmatrix} 1 & 0 \\ 0 & 3/2 \end{bmatrix} \qquad 3.2\text{-}21$$

produces

$$\underline{L}'' = \begin{bmatrix} 2 & 3/2 \\ 3/2 & 9/4 \end{bmatrix} \quad \text{and} \quad \underline{S}'' = \begin{bmatrix} 0 & 0 \\ 0 & 9/4 \end{bmatrix} \qquad 3.2\text{-}22$$

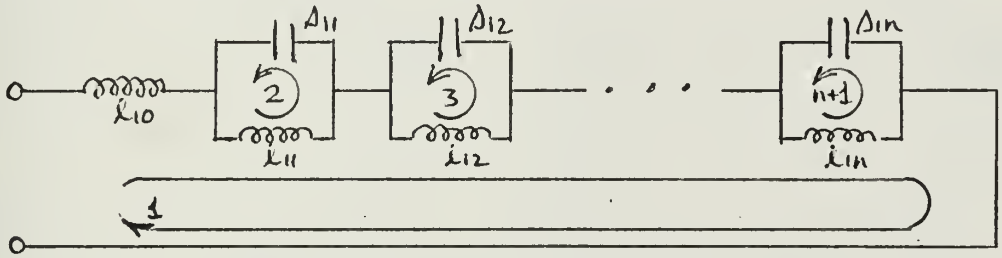
These matrices form the circuit below.



Thus, there are an infinity of realizable circuits with the impedance function 3.2-20 above that can be found via Howitt's transformation.

3.3 APPLICATION TO GENERAL FOSTER FORMS

In section 2.5, Kalman's transformation was applied to the general state equations for the 2n+1 order Foster LC circuits, and the transformation matrix was found. The matrices were necessarily partitioned into three parts, of 1, n, and n. In the same circuit, however, there are only n+1 loops to be considered when using Howitt's transformation. This simplifies the calculations tremendously, and allows the matrices to be partitioned in only two parts. As in the Kalman transformation, though, the complete solution is again prohibited by a set of m equations to find m variables in which more than one variable appears in each equation.



The Impedance-form Foster

If the loops in the above circuit are taken to be as shown, the $\underline{L}$ and $\underline{S}$ matrices partition very easily. The sum of the inductances in the first loop is simply the sum of all the inductances in the circuit. Each inductance is common to the first loop and its own loop, so the inductance matrix is

$$\underline{L} = \begin{bmatrix} \sum_{i=0}^{n} l_{1i} & l_{11} & l_{12} & \cdots & l_{1n} \\ l_{11} & l_{11} & 0 & & 0 \\ l_{12} & 0 & l_{12} & & 0 \\ \vdots & & & \ddots & \vdots \\ l_{1n} & 0 & 0 & \cdots & l_{1n} \end{bmatrix} \qquad 3.3\text{-}1$$

which can be partitioned as

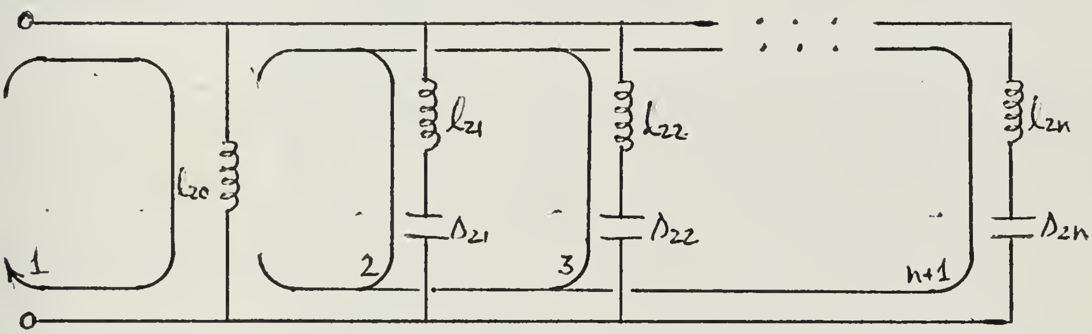
$$\underline{L} = \begin{array}{cc} & \begin{array}{cc} 1 & n \end{array} \\ \begin{array}{c} 1 \\ n \end{array} & \left[\begin{array}{c|c} \sum_{i=0}^{n} l_{2i} & L_1^{(r)} \\ \hline L_1^{(c)} & L_1 \end{array}\right] \end{array} \qquad 3.3\text{-}2$$

The susceptances are each contained solely in their own loops, so the susceptance matrix is

$$\underline{S} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & s_{11} & 0 & & 0 \\ 0 & 0 & s_{12} & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & s_{1n} \end{bmatrix} \qquad 3.3\text{-}3$$

which can be partitioned in the same way as

$$\underline{S} = \begin{array}{cc} & \begin{array}{cc} 1 & n \end{array} \\ \begin{array}{c} 1 \\ n \end{array} & \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & S_1 \end{array}\right] \end{array} \qquad 3.3\text{-}4$$



The Admittance-form Foster

The admittance-form Foster circuit presents a serious problem unless the loops are defined somewhat strangely. If the meshes are taken as loops, the matrices found cannot be partitioned at all, so the method above is used. This does not affect the transformation, because this definition still has just the first loop at the port so that the loop current and voltage of this loop can be maintained without affecting the others.

With the loops as shown, the sum of inductances in the

first loop is simply l_{20}, but this is also common to every loop. Each of the loops other than the first has a sum of inductances equal to l_{20} plus the inductance in the corresponding branch. Thus, the inductance matrix is

$$\underline{L}' = \begin{bmatrix} l_{20} & l_{20} & l_{20} & \cdots & l_{20} \\ l_{20} & (l_{20}+l_{21}) & l_{20} & & l_{20} \\ l_{20} & l_{20} & (l_{20}+l_{22}) & & l_{20} \\ \vdots & & & & \vdots \\ l_{20} & l_{20} & l_{20} & \cdots & (l_{20}+l_{2n}) \end{bmatrix} \qquad 3.3\text{-}5$$

which can again be partitioned the same way as

$$\underline{L}' = \begin{array}{c|c} & \begin{array}{cc} 1 & n \end{array} \\ \begin{array}{c} 1 \\ n \end{array} & \left[\begin{array}{c:c} l_{20} & l_{20}^{(r)} \\ \hdashline l_{20}^{(c)} & (l_{20}^{(n)} + L_2) \end{array}\right] \end{array} \qquad 3.3\text{-}6$$

where $l_{20}^{(n)}$ indicates an nXn matrix of l_{20}'s. The susceptance matrix is as simply written, as

$$\underline{S}' = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & \Delta_{21} & 0 & & 0 \\ 0 & 0 & \Delta_{22} & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \Delta_{2n} \end{bmatrix} \qquad 3.3\text{-}7$$

and partitioned as

$$\underline{S}' = \begin{array}{c|c} & \begin{array}{cc} 1 & n \end{array} \\ \begin{array}{c} 1 \\ n \end{array} & \left[\begin{array}{c:c} 0 & 0 \\ \hdashline 0 & S_2 \end{array}\right] \end{array} \qquad 3.3\text{-}8$$

The transformation matrix is assumed partitioned in the same manner

$$\underline{A} = \begin{array}{c|c} & \begin{array}{cc} 1 & n \end{array} \\ \begin{array}{c} 1 \\ n \end{array} & \left[\begin{array}{c:c} 1 & 0 \\ \hdashline A_{21} & A_{22} \end{array}\right] \end{array} \qquad 3.3\text{-}9$$

and is applied to the partitioned matrices in equations 3.3-2, 4, 6, and 8:

$\boxed{\underline{L}' = \underline{A}^t \underline{L} \underline{A}}$ becomes

$$\begin{bmatrix} l_{20} & l_{20}^{(r)} \\ l_{20}^{(c)} & (l_{20}^{(n)}+L_2) \end{bmatrix} = \begin{bmatrix} \left(\sum_{i=0}^{n} l_{1i} + L_1^{(r)}A_{21} + A_{21}^t L_1^{(c)} + A_{21}^t L_1 A_{21}\right) & \left(L_1^{(r)}A_{22} + A_{21}^t L_1 A_{22}\right) \\ \left(A_{22}^t L_1^{(c)} + A_{22}^t L_1 A_{21}\right) & \left(A_{22}^t L_1 A_{22}\right) \end{bmatrix} \qquad 3.3\text{-}10$$

$\boxed{\underline{S}'=\underline{A}^t\underline{S}\,\underline{A}}$ becomes

$$\begin{bmatrix} 0 & 0 \\ 0 & S_2 \end{bmatrix} = \begin{bmatrix} A_{21}^t S_1 A_{21} & A_{21}^t S_1 A_{22} \\ A_{22}^t S_1 A_{21} & A_{22}^t S_1 A_{22} \end{bmatrix} \qquad 3.3\text{-}11$$

Six equations are obtained from equations 3.3-10 and 11 which will be used to solve for the elements of $\underline{A}$ and the constraints. Only one is dependent, equation 3.3-16.

3.3-12 $l_{20}=\sum_{i=0}^{n} l_{ii}+L_1^{(r)}A_{21}+A_{21}^t L_1^{(c)}+A_{21}^t L_1 A_{21}$ 3.3-13 $l_{20}^{(c)}=A_{22}^t L_1^{(c)}+A_{22}^t L_1 A_{21}$

3.3-14 $l_{20}^{(n)}+L_2=A_{22}^t L_1 A_{22}$ 3.3-15 $0=A_{21}^t S_1 A_{21}$

3.3-16* $0=A_{22}^t S_1 A_{21}$ 3.3-17 $S_2=A_{22}^t S_1 A_{22}$

Becuase S_1 is diagonal, the equation 3.3-15 produces

$$0=\sum_{i=1}^{n}(a_{i1}^{21})^2 \Delta_{1i} \qquad 3.3\text{-}18$$

and since this must be true for any set of Δ_{1i}'s, then

$$A_{21}=0 \qquad 3.3\text{-}19$$

and equation 3.3-16 is dependent. In equation 3.3-17, the form of the off-diagonal terms is

$$0=\sum_{k=1}^{n} a_{ki}^{22} a_{kj}^{22} \Delta_{1k}, \quad i\neq j \qquad 3.3\text{-}20$$

and the form of the main diagonal terms is

$$\Delta_{2i}=\sum_{k=1}^{n}(a_{ki}^{22})^2 \Delta_{1k} \qquad 3.3\text{-}21$$

If an assumed possible form of solution is $a_{ij}^{22}=0$ for $i\neq j$, which satisfies equation 3.3-20, then equation 3.3-21 produces

$$\Delta_{2i}=(a_{ii}^{22})^2\Delta_{1i} \quad \text{or} \quad a_{ii}^{22}=\sqrt{\frac{\Delta_{2i}}{\Delta_{1i}}} \qquad 3.3\text{-}22$$

Unfortunately, the off-diagonal terms of equation 3.3-14 are of the form

$$l_{20}=\sum_{k=1}^{n} a_{ki}^{22} a_{kj}^{22} l_{1k}, \quad i\neq j \qquad 3.3\text{-}23$$

and the assumption above does not satisfy this equation.

This type of solution is thus not possible, and the equations left cannot be solved in general terms, with the exception of equation 3.3-12, which has the solution

$$l_{20} = \sum_{i=0}^{n} l_{1i} \qquad 3.3\text{-}24$$

The remaining equations can be used to determine the general form of the equations to be solved in a specific case.

From equation 3.3-13,

$$l_{20} = \sum_{i=1}^{n} a_{ki}^{22} l_{1i} \,, \quad k = 1 \text{ to } n \qquad 3.3\text{-}25$$

Finally, the main diagonal terms of equation 3.3-14 are of the form

$$l_{20} + l_{2i} = \sum_{k=1}^{n} (a_{ki}^{22})^2 l_{1k} \,, \quad i = 1 \text{ to } n \qquad 3.3\text{-}26$$

From the above equations, the form of the transformation matrix is

$$\underline{A} = \begin{array}{c} \\ 1 \\ n \end{array} \!\!\begin{array}{c} \begin{array}{cc} 1 & n \end{array} \\ \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & A_{22} \end{array}\right] \end{array} \qquad 3.3\text{-}27$$

where the elements of A_{22} and the constraints are found from the equations 3.3-20, 21, 23, 25, and 26.

Even though the solution is carried no further in this section than it was in section 2.5, the point to be drawn here is that the calculations involved were much simpler than in Kalman's transformation. Of course, in this particular problem, the state equations are of dimension three while the loop equations are of dimension two. It is conceivable that another problem could find the exact opposite situation. Nevertheless, it is still true that it is not necessary to find one entire row (or more

in dealing with transfer functions) of the transformation matrix of Howitt's transformation while the entire matrix must be found in Kalman's transformation. This ensures that, even in an equal case, the complexity of the equations in Howitt's transformation will be considerably less than that of Kalman's transformation.

3.4 EXAMPLE: Brune and Bott-Duffin Synthesis Circuits

In section 2.4, it was shown that some possibly equivalent circuits cannot be handled by Kalman's transformation. The circuits used to demonstrate were the Brune and Bott-Duffin basic synthesis forms. A careful examination of the circuit diagrams in that section (pages 21 and 24, respectively) will show that Howitt's transformation likewise cannot be used, for the simple reason that the two circuits are not topologically equivalent. Brune's circuit has two loops, while the Bott-Duffin circuit has three. One of the basic assumptions in Howitt's transformation is that the circuits used must be of the same order in the loop equations, so the transformation cannot be used.

IV. CONCLUSIONS

The two transformations dealt with in this thesis are both designed to be used for developing many circuits with the same characteristics, yet they attack the problem from two cifferent directions and the results are quite different.

Kalman's transformation is based on the state equations of the different circuits. Through manipulation of the matrices of the state equations, the transformation manages to keep the transfer function from input to output invariant. The basic restriction on this first transformation is that the direct relation between input and output must be the same in all circuits. This seriously hampers the field of endeavor, excluding a great number of equivalent circuits in which the optimum circuit may lie. In addition, in most cases the complexity of the transformation is such that solution of the problem is made very difficult. On the other hand, the transformation can be a very powerful tool in some of the simpler cases when it is desired to find the constraints between the two circuits' elements. One point discourages the actual finding of a multitude of circuits with Kalman's transformation: the difficulty of finding the proper circuit from the state equations found.

Conversely, Howitt's transformation uses the loop impedance matrices, and the step from a generated loop

impedance to the corresponding circuit is very simple. This transformation deals directly with the impedances, altering these matrices to maintain the current and voltage in one or more loops, and consequently the input impedance to those loops. The main restriction here is that a great number of the circuits generated at random by this transformation would contain negative elements, undesirable for most circuits. By a careful examination of the requirements on the new impedance matrices for positive elements, one can find the range of transformation matrices which will produce realizable circuits, but the mathematics of this approach is fairly prohibitive. Despite this restriction, Howitt's transformation can handle fairly complex circuits with ease, compared to the unwieldy Kalman's.

This has not been an exhaustive examination of the two transformations, but the applications and limitations of each have been shown and, using the transformations, sections 2.5 and 3.3 provide the basis for ease in translating from the impedance-form Foster circuit to the admittance-form and vice versa. The procedure used in those two sections can be applied to a wide range of possibly equivalent circuit forms to obtain satisfactory results.

BIBLIOGRAPHY

[1] Anderson, B.D.O., R.W. Newcomb, R.E. Kalman, and D.C. Youla, "Equivalence of Linear Time-Invariant Dynamical Systems," Journal of the Franklin Institute, vol. 281, pp. 371-378; May 1966.

[2] Ardalan, A., and S.R. Parker, "On the Growth of Equivalent Networks with Imposed Design Constraints," Proceedings of the First Annual Princeton Conference on Information Sciences and Systems, Princeton, N.J.; March 1967.

[3] Brune, O. "Synthesis of a Finite Two Terminal Network Whose Driving Point Impedance is a Prescribed Function of Frequency," J. Math. Phys., vol. 10, pp. 191-236; August 1931.

[4] Cauer, W., Synthesis of Linear Communications Networks, vols. 1 and 2, p. 194, McGraw-Hill Book Co., Inc, 1958.

[5] Howitt, Nathan, "Group Theory and the Electric Circuit," The Physical Review, vol. 37, pp. 1583-1595; June 15, 1931.

[6] Kalman, R.E., and B.L. Ho, "Effective Construction of Linear State-Variable Models from Input/Output Data," 1965 Allerton Conference Proceedings.

[7] Karni, Shlomo, Network Theory: Analysis and Synthesis, pp. 145-151, 166-180, Allyn & Bacon, Inc., 1966.

[8] Newcomb, Robert W., Active Integrated Circuit Synthesis, Prentice-Hall Inc., c.1968.

[9] Schoeffler, J.D., "Continuously Equivalent Networks and Their Applications," IEEE Transactions on Communications and Electronics, vol. 83, pp. 763-767; November 1964.

[10] Schoeffler, "The Synthesis of Minimum Sensitivity Networks," IEEE Transactions on Circuit Theory, vol. CT-11, pp. 271-276; June 1964.

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13. ABSTRACT

Kalman's and Howitt's equivalence transformations are applied to the canonic impedance and admittance Foster LC forms and the Cauer ladder realizations for an RC circuit. The results provide a format for transforming from one realization to another directly. Application of the Kalman transformation to second-order Brune and Bott-Duffin realizations indicates that they are not compatible, implying the incompleteness of Kalman's transformation theory. The same technique is used to show a similar incompleteness of Howitt's theory.

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